Title: Sign-changing self-similar solutions of the nonlinear heat equation with positive initial value

Abstract: In this talk, I present a joint work with Flávio Dickstein, Ivan Naumkin and Fred B. Weissler. We consider the nonlinear heat equation $u_t - \Delta u = |u|^\alpha u$ on $\mathbb{R}^N$, where $\alpha > 0$. It is well known that the Cauchy problem is locally well-posed in a variety of spaces. On the other hand, the Cauchy problem is not well posed for initial values which are too singular. In particular, it is known that if $u_0 \geq 0$ and $u_0(x) \geq \mu |x|^{-\frac{2}{\alpha}}$ in a neighborhood of the origin, with $\mu > 0$ sufficiently large, then there is no local, nonnegative solution of the Cauchy problem with initial condition $u(0,x) = u_0(x)$, in any reasonable sense. We prove that in the range $0 < \alpha < \frac{4}{N-2}$, for every $\mu > 0$, there exist infinitely many self-similar solutions to the Cauchy problem with initial value $u_0(x) = \mu |x|^{-\frac{2}{\alpha}}$. Of course, these solutions are sign-changing if μ is sufficiently large. The construction is based on the analysis of the related inverted profile equation. These solutions can be perturbed, and we show that if $u_0(x) = \mu \rho(x) |x|^{-\frac{2}{\alpha}}$ where ρ is a cut-off function around 0, then there exist infinitely many local solutions to the Cauchy problem with initial value u_0. This yields a whole class of nonnegative initial values for which there is no nonnegative local solution, but for which there exist infinitely many sign-changing solutions.